ANOMALOUS NOISE PEAK IN ADLER TUBES

BY

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ABSTRACT

An experimental investigation of the noise stripping mechanism in two Adler tubes with high residual gas pressure (> 10^{-6} mm Hg) has revealed the presence of an anomalous noise peak (> 1 000 $^{\circ}$K). The peak occurs at a frequency somewhat lower than the cyclotron frequency. It is proposed that the peak is produced by elastically scattered beam electrons traveling slowly through the input coupler. The phenomenon is described in terms of an idealized theory which is in good agreement with the experimental findings.

I. INTRODUCTION

This report deals with noise generated by scattered beam electrons in Adler tubes. Specifically the noise originating from electrons scattered by neutral gas molecules (imperfect vacuum) is considered (chapter II). It is realized, however, that under certain conditions other types of beam electron scattering, e.g. scattering from the beam forming electrodes, may also contribute to the phenomenon which will be reported.

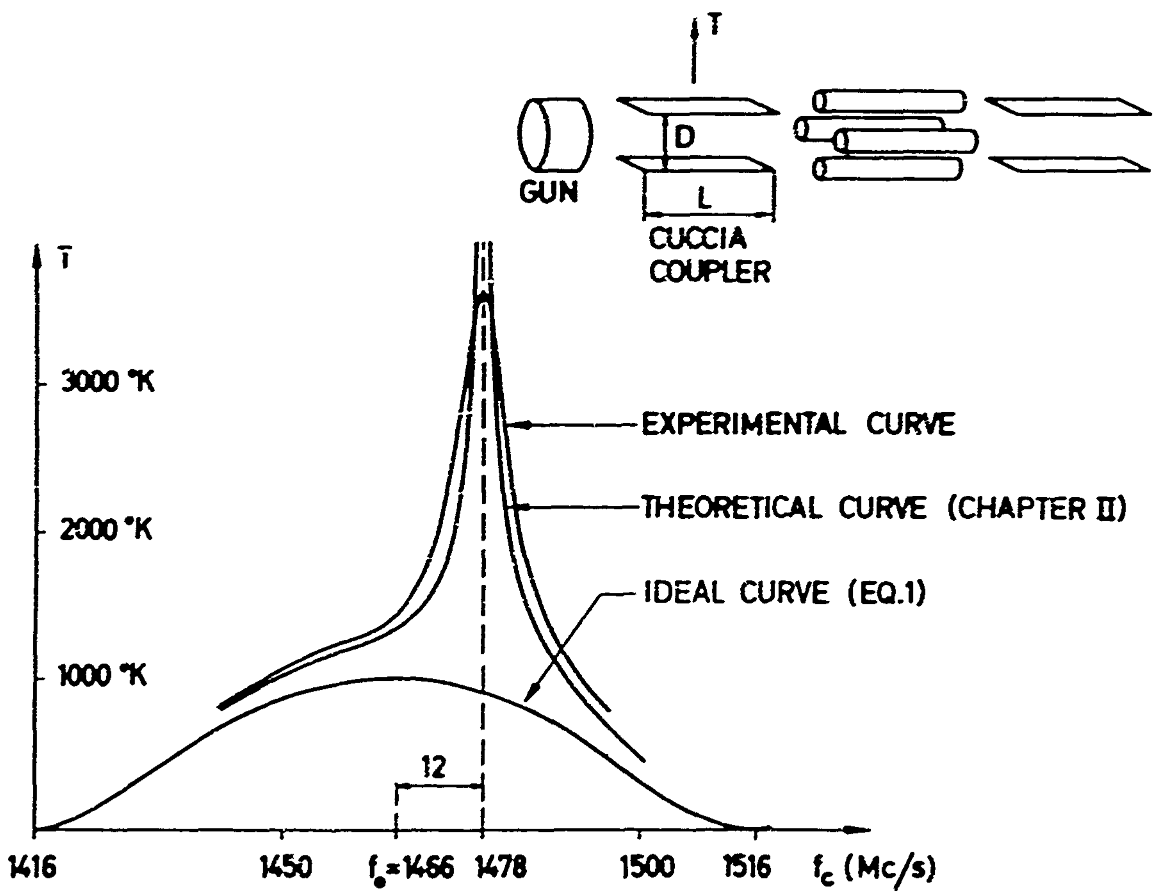

Fig. 1. Noise temperature attributable to the electron beam versus the cyclotron frequency. The temperature refers to the terminals of the matched fixed frequency receiver connected to the input Cuccia coupler in an Adler tube.

The phenomenon - the anomalous noise peak in Fig. 1 - revealed itself in connection with an experimental investigation of the noise stripping mechanism in two Adler tubes built at this laboratory for radio astronomy use. The noise extracted by the input Cuccia coupler as a function of the cyclotron frequency (i.e. the dc magnetic field) was recorded in the course of the experiments. The receiver was matched to the beam-loaded input Cuccia coupler. The quadrupole, which was unpowered, and the output Cuccia coupler can be disregarded in this particular experiment. The receiver (bandwidth 1 Mc/s) was tuned to the resonance frequency of the coupler (f_o = 1466 Mc/s). The latter had a bandwidth of about $\pm$ 30 Mc/s. In an ideal Cuccia coupler [1] one would expect to measure the noise temperature

$$T_n = T_c \frac{4GG_o}{\left|G_o+G+jB\right|^2} \tag{1}$$

which is plotted in Fig. 1 as the expected curve. The notations are:

T_c = fast cyclotron wave temperature $\simeq$ cathode temperature,

$$G+jB = G_o \left[\left(\frac{\sin\frac{\Delta\omega_c\tau_o}{2}}{\frac{\Delta\omega_c\tau_o}{2}}\right)^2 - j\frac{2}{\Delta\omega_c\tau_o}\left(1-\frac{\sin\Delta\omega_c\tau_o}{\Delta\omega_c\tau_o}\right)\right] =$$

= electronic admittance of the Cuccia coupler,

$G_o = \frac{1}{8}\frac{I_o}{V_o}\left(\frac{L}{D}\right)^2$ = electronic admittance of the Cuccia coupler at resonance ($\Delta\omega_c = 0$) = external matching load (receiver),

$\Delta\omega_c = \omega_o - \omega_c$; $\omega_o = 2\pi f_o$; ω_c = angular cyclotron frequency,

τ_o = L/v_o = transit time through the coupler,

L = length of coupler, D = distance between coupler electrodes,

v_o = beam velocity, V_o = accelerating voltage, I_o = beam current.

The measured curve is characterized by a noise peak. The peak always appeared at a cyclotron frequency value which was higher than the measuring frequency, quite typically by 12 Mc/s as shown in Fig. 1. The noise peak has also been observed by R. Adler [2] in tubes with imperfect vacuum (10^{-6} mm Hg).

The proposed explanation of the anomalous noise peak is the following. Some of the electrons in the beam are elastically scattered by neutral gas molecules (the beam voltage is lower than the ionization potentials). The electrons scattered at nearly right angles travel slowly in the axial direction and spend a comparatively long time in the coupler. The transverse energy of these electrons (in eV) equals the beam voltage (typically 3-10 V corresponding to a kinetic temperature range of about 30 000 - 100 000°K). Even if the number of such electrons is small, the considerable time they spend in the coupler and the high temperatures they represent could easily account for the 3 000°K peak in Fig. 1.

In vacuum the scattered electrons would rotate with the cyclotron frequency. In the case considered here the scattered electrons rotate in the space charge of the unscattered electron beam. According to [3] the frequency of rotation is, if $(f_c - f_r)/f_c \ll 1$,

$$f_r = \frac{f_c}{2} + \frac{f_c}{2}\sqrt{1 - 2\left(\frac{f_p}{f_c}\right)^2} \approx f_c - \frac{1}{2}\frac{f_p^2}{f_c} \qquad (2)$$

where f_p is the plasma frequency[x)] of the beam.

The noise peak should appear when the rotational frequency f_r coincides with the receiver frequency f_o, i.e. (Eq. 2) when

$$f_c - f_o \approx \frac{1}{2} f_p^2 / f_c \qquad (3)$$

Indeed, the experiments indicate that the measured frequency difference $f_c - f_o$ varies linearly with the estimated beam electronic charge density (beam current and beam voltage were varied). Furthermore, in the case shown in Fig. 1 the beam current was 14 μA and the beam voltage 3V (corrected for contact potentials) while the measured $f_c - f_o$, according to Fig. 1, was 12 Mc/s. Using this information in Eq. (3) one can solve for the beam radius which turns out to be 0.25 mm. This is in very good agreement with the estimated beam radius (0.2-0.3 mm).

The peak appears because the important electrons with long transit times through the Cuccia coupler and with high transverse energies produce a strong but narrow spectrum around f_r.

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x) $f_p^2 = \frac{1}{4\pi^2} \frac{e}{m} \frac{|\rho_o|}{\varepsilon_o}$ where $\frac{e}{m}$ is the electronic charge to mass ratio ($= 1,76 \cdot 10^{11}$ As/kg), ρ_o the volume density of charge in the beam and ε_o the dielectric constant in vacuum.

The proposed mechanism thus gives a qualitatively very satisfactory explanation to the observed anomalities. A quantitative although highly idealized theory is presented in the next chapter. The purpose of the theory is to calculate the equivalent input noise temperature due to the scattered electrons in the input Cuccia coupler.

II. ELEMENTARY THEORY OF SCATTERED ELECTRON INDUCED NOISE IN CUCCIA COUPLERS

The electron beam current I_o in a typical low noise Adler tube is 10 - 100 μA while the accelerating voltage V_o (Cuccia coupler voltage, corrected for contact potentials) is 3 - 10 V. The cathode to collector distance is of the order 10 cm while the length, L, of each Cuccia coupler is 1 - 2 cm.

In the presence of residual gas in the tube some of the electrons will be scattered by neutral gas molecules. The scattered current I_s can be written

$$\frac{I_s}{I_o} = \sigma n Z \tag{4}$$

where

σ = average scatter cross section per molecule ($\simeq 3 \cdot 10^{-19}$ m^2 in the electron velocity range of interest [4])

n = number density of neutral gas molecules ($\simeq 3 \cdot 10^{22}$ m^{-3} at room temperature and at a pressure of 1 torr)

Z = effective distance (along the beam) which contributes with scattered electrons to the coupler ($\simeq 10^{-1}$ m)

At a pressure of 1 μtorr (10^{-6} mm Hg) one obtains, using these values in Eq. (4), $I_s/I_o \simeq 10^{-3}$.

We will adopt the assumption that the scattering is iso-

tropic. The current element dI_s associated with the scattered electrons in the axial velocity range $v \pm \frac{1}{2} dv$ then becomes

$$dI_s = \frac{I_s}{2 v_o} dv \qquad (5)$$

Note that dI_s/dv is independent of v.

The electronic admittance, $g + jb$, of the scattered electrons traversing the coupler is negligibly small compared to the admittance, $G + jB$, presented by the main beam. The latter equals G_o if $\Delta\omega_c L/v_o << \pi/2$ which will be assumed in what follows.

We will now consider an element dI_s of the scattered beam in the coupler. The electronic conduciance element associated with the current element dI_s (axial velocity v) is

$$dg = \frac{1}{8} \frac{dI_s}{\left(\frac{m v^2}{2 e}\right)} \left(\frac{L}{D}\right)^2 \left[\frac{\sin\left(\frac{\Delta\omega_r}{2} \frac{L}{v}\right)}{\left(\frac{\Delta\omega_r}{2} \frac{L}{v}\right)}\right]^2 \qquad (6)$$

according to well known Cuccia coupler equations [1]. Here $\Delta\omega_r = \omega_o - \omega_r$ while ω_r is the angular rotation frequency of the scattered electrons (Eq. 2). The scattered electrons rotate with a radius of

$$r \simeq \frac{\sqrt{v_o^2 - v^2}}{\omega_c} \lesssim \frac{v_o}{\omega_c} \simeq \frac{\sqrt{V_o\,(\text{volt})}}{10\, f_c\,(\text{kMc/s})} \text{ mm} \qquad (7)$$

i.e. $r_{max} \simeq 0.15$ mm when $V_o = 5V$ and $f_c = 1.5$ kMc/s. The beam radius is usually larger which means that the electrons rotate in the space charge field. The latter counteracts the magnetic force thereby reducing the frequency of rotation.

The center of gravity of the scattered beam will rotate inside the main beam with the same frequency as the individual electrons, i.e. with the frequency f_r rather than with the main beam rotational frequency f_c. It should be noted that we consider the main beam and the scattered beam as two different things. The reason fo· doing so is that the quadrupole, if it is pumped, would intercept the transversely energetic scattered electrons and the main beam alone would reach the output coupler. A common center of gravity analysis for the main and scattered beams therefore is unrealistic. It is also clear that under these circumstances the usual noise stripping mechanism does not work as far as the noise transferred from the scattered electrons via the input coupler to the main beam is concerned.

The equivalent transverse noise temperature $T_e(v)$ of the scattered beam element dI_s is given by

$$T_e(v) = \frac{m}{2k}(v_o^2 - v^2) \equiv T_o(1 - \frac{v^2}{v_o^2}) \qquad (8)$$

where k = Boltzmann's constant = $1.38 \cdot 10^{-23}$ Ws/^oK and $T_o = mv_o^2/(2k) = eV_o/k$.

We can now consider the conductance element dg as a resistive noise current generator of temperature $T_e(v)$ connected across the Cuccia coupler. The latter is loaded with the parallel combination of the main beam (conductance G_o) and the matched receiver (same conductance). The noise temperature T_s, due to the scattered beam, measured by the receiver is

$$T_s = \int \frac{T_e}{G_o} dg \qquad (9)$$

or, with Eq.s (5), (6) and (8) and with the expression for G_o on p. 3

$$\frac{T_s}{T_o} = \frac{1}{2}\frac{I_s}{I_o}\int_o^{v_o}\left(1-\frac{v^2}{v_o^2}\right)\left(\frac{v_o}{v}\right)^2\left[\frac{\sin\frac{\Delta\omega_r L}{2v}}{\frac{\Delta\omega_r L}{2v}}\right]^2\frac{dv}{v_o} \qquad (10)$$

In Eq. (10) positive v is thought to represent electrons which move towards the coupler after the scattering has occurred while electrons with negative v move away from it. This explains the choice of the integration limits in Eq. (10).

With the substitution $u = \frac{1}{2}\Delta\omega_r L/v = \frac{1}{2}\Delta\omega_r \tau$ one obtains

$$\frac{T_s}{T_o} = \frac{1}{2}\frac{I_s}{I_o}\frac{1}{u_o}\int_{u_o}^{\infty}\left(1-\frac{u_o^2}{u^2}\right)\left(\frac{\sin u}{u}\right)^2 du \qquad (11)$$

where $u_o = \frac{1}{2}\Delta\omega_r L/v_o = \frac{1}{2}\Delta\omega_r\tau_o$. If $u_o \ll \pi/2$, which is usually the case, we can write approximately

$$\frac{T_s}{T_o} \simeq \frac{1}{2}\frac{I_s}{I_o}\frac{1}{u_o}\int_o^{\infty}\left(\frac{\sin u}{u}\right)^2 du = \frac{1}{4}\frac{I_s}{I_o}\frac{\pi}{u_o} \qquad (12)$$

or

$$\boxed{T_s \simeq \frac{T_o}{4}\frac{I_s}{I_o}\frac{\pi}{u_o} = \frac{T_o}{4}\frac{I_s}{I_o}\frac{1}{\Delta f_r\tau_o}} \qquad (13)$$

where $\Delta f_r = f_o - f_r = \Delta\omega_r/2\pi$. Eq. (13) describes the noise temperature measured (as a function of frequency) at the input Cuccia coupler in excess of the ordinary noise temperature T_n described by Eq. (1). T_s is also the noise temperature (due to scattered electrons) which will be imposed on

the main beam at the input Cuccia coupler in the form of fast cyclotron wave noise. T_s is proportional to the beam voltage and the residual gas pressure and inversely proportional to the difference between the measuring frequency and the rotational frequency of the scattered electrons.

In the experiment described in Chaper I the data were as follows

$$T_o \simeq 30\,000\ ^{o}K\ \ (V_o = 3V);$$

$I_s/I_o \simeq 10^{-2}$ (estimated pressure $5 \cdot 10^{-6}$ mm Hg, estimated effective beam length $Z \simeq 0.2$ m)

and $\tau_o = 2 \cdot 10^{-8}$ s

which gives

$$T_s \simeq \frac{4\,000}{\Delta f_r (Mc/s)}\ \ ^{o}K$$

The sum of T_s, according to this expression, and T_n, according to Eq. (1), is shown in Fig. 1. It can be concluded that the theory satisfactorily explains the experimental result.

Eq. (13) has a singularity when $\Delta f_r = 0$. This is due to the fact that the scattered electrons with axial velocity $v = 0$ would, according to the previous mathematics, spend an infinite time in the coupler. If we introduce an upper time limit τ_{max} ($\gg \tau_o$) and consider only small Δf_r assuming $\frac{1}{2}\Delta\omega_r \tau_{max} \ll \frac{1}{2}\pi$, we can write Eq. (12) approximately

$$\frac{T_s}{T_o} \simeq \frac{1}{2}\frac{I_s}{I_o}\frac{1}{u_o}\int_o^{\frac{1}{2}\Delta\omega_r\tau_{max}} du \qquad = \frac{1}{2}\frac{I_s}{I_o}\frac{\tau_{max}}{\tau_o} \tag{14}$$

or

$$\boxed{T_s \simeq \frac{T_o}{2}\frac{I_s}{I_o}\frac{\tau_{max}}{\tau_o}} \tag{15}$$

which is the (finite) temperature of the noise peak ($\Delta f_r = 0$). With the data of the experiment Eq. (15) yields

$$T_s = 150\,\frac{\tau_{max}}{\tau_o}\ {}^oK$$

Since the experimental peak temperature was about 2 500 ^oK one obtains $\tau_{max}/\tau_o \simeq 15$ which is reasonable with regard to expected inhomogeneities in the axial dc potential etc.

The following practical conclusions concerning low noise Adler tubes are obvious:

1) The residual gas pressure must be well below 10^{-7} mm Hg

2) The dc voltages should be adjusted so as to sweep away the axially slow, scattered electrons from the input Cuccia coupler

3) The charge density of the beam should be large in order to keep the noise peak as far as possible from the center of the signal transmission frequency band. For the same reason the cyclotron frequency should be adjusted to a slighly lower value than the center frequency of the band.

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